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(2 + 1)-dimensional compacton solutions with and without completely elastic interaction properties

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Abstract
Taking the (2 + 1)-dimensional Broer–Kaup–Kupershmidt system as a simple example, some special types of (2 + 1)-dimensional compacton solutions are constructed. It is shown that there is quite rich interaction behaviour between two travelling compactons. For some types of compactons, the interactions among them may not be completely elastic. For some others, the interactions are completely elastic. There is no phase shift for the interactions of the (2 + 1)-dimensional compactons discussed in this paper.

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1. Introduction

Recently, in addition to the peakon solutions (a special type of weak solution of the (1 + 1)-dimensional Camassa–Holm (CH) equation [1]), some other types of weak solutions in nonlinear systems have attracted much attention. Among them, the so-called compacton solution is one of the most important excitations. Compacton solutions describe the typical (1 + 1)-dimensional soliton solutions with finite wavelength when the nonlinear dispersion effects are included in the models [2] and may have many interesting properties and possible physical applications [3–5]. For instance, the compacton equations may be used to study the motion of ion-acoustic waves and a flow of a two-layer liquid [4]. In [5], the Painlevé integrability of two sets of Korteweg–de Vries (KdV) type and modified KdV-type compacton equations are proved.

In this short paper, we try to find some kinds of compacton solutions in (2 + 1) dimensions. Especially, we are interested in the possible interaction behaviour among (2 + 1)-dimensional compactons.

To study the (2 + 1)-dimensional compactons, we take the (2 + 1)-dimensional Broer–Kaup–Kupershmidt (BKK) system

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$$H_{ty} - H_{xxy} + 2(HH_x)_y + 2G_{xx} = 0 \tag{1}$$

$$G_t + G_{xx} + 2(HG)_x = 0 \tag{2}$$

as a simple illustration. The BKK system may be derived from the inner parameter-dependent symmetry constraint of the Kadomtsev–Petviashvili (KP) equation [6]. Though the integrability of the BKK system can be guaranteed by the integrability of the KP equation (because it is a *symmetry* constraint of the KP equation), some authors have strictly proved its integrability in some different sense. For instance, the Painlevé integrability and infinitely many symmetries of the model have been given in [7]. Using some suitable dependent and independent variable transformations [8], Chen and Li have proved that the BKK system can be transformed to the (2 + 1)-dimensional integrable dispersive long wave equation system (DLWE)

$$u_{ty} = -\eta_{xx} - \tfrac{1}{2}(u^2)_{xy} \tag{3}$$

$$\eta_t = -(u\eta + u + u_{xy})_x \tag{4}$$

and the (2 + 1)-dimensional integrable AKNS (Ablowitz–Kaup–Newell–Segur) system

$$\psi_t = -\psi_{xx} + \psi u \tag{5}$$

$$\phi_t = \phi_{xx} - \phi u \tag{6}$$

$$u_y = \psi\phi. \tag{7}$$

The 'weak' Lax pair and the inverse scattering scheme of the (2 + 1)-dimensional DLWE have been given by Boiti *et al* [9].

When we take $y = x$, the BKK system (1) and (2) is reduced to the usual (1 + 1)-dimensional BKK system, which can be used to describe the propagation of long waves in shallow water [10]. Some authors have given many kinds of special solutions of the (2 + 1)-dimensional BKK system, DLWE system and AKNS system [11–14]. Especially, Ying and Lou had obtained some coherent structures of the BKK equation by means of the truncated Painlevé expansion and the related Bäcklund transformation [11]. A somewhat richer class of solutions of the BKK system can be found by means of the variable separation approach [12]. In [12], a special solution with some arbitrary functions is given. By selecting the arbitrary functions appropriately, rich continuous localized solutions and the peakon solutions of the BKK system have been discussed also in [12].

In section 2 of this paper, a short review on the variable separation procedure for the BKK system is described. The compacton solutions and their interaction properties are discussed in section 3. The last section is a short summary and discussion.

2. Review of the variable separation procedure

To study the possible compacton excitations, we give a short review of the variable separation procedure here especially for the BKK system.

The first step of our variable separation approach is to change the original model to a general multi-linear form by using the Painlevé Bäcklund transformation. For the BKK system the corresponding Painlevé Bäcklund transformation reads

$$\begin{cases} H = (\ln f)_x + H_0 \\ G = (\ln f)_{xy} + G_0 \end{cases} \tag{8}$$

where $\{H_0, G_0\}$ is an arbitrary known seed solution of the BKK system.

Substituting (8) into the first equation of the BKK system leads to the following trilinear form:

$$2H_0\left(2f_x^2f_y+f^2f_{xxy}-ff_yf_{xx}-2ff_xf_{xy}\right)+2H_{0x}f(ff_{xy}-f_xf_y)+2f^2f_xH_{0xy}$$
$$-f(f_xf_{ty}+f_yf_{tx}+f_tf_{xy}+f_xf_{xxy}+f_yf_{xxx}+f_{xx}f_{xy})+f^2(f_{txy}+f_{xxxy})$$
$$+2f_xf_y(f_t+f_{xx})+2H_{0y}f\left(ff_{xx}-f_x^2\right)=0. \qquad (9)$$

Using relations (9) and (8), the second equation of the BKK system is simplified to a bilinear equation

$$\left(ff_{xx}-f_x^2+ff_x\partial_x\right)(G_0-H_{0y})=0. \qquad (10)$$

The second step of the variable separation approach first developed in [16] is to select a suitable trivial seed solution with arbitrary function(s). For the BKK system, it is easy to see that

$$H_0\equiv h_0=h_0(x,t)\qquad G_0=0 \qquad (11)$$

with h_0 being an arbitrary function of $\{x,t\}$ is a trivial solution of the BKK system. For the seed solution (11), equation (10) satisfies identically.

The next step is to solve the multi-linear equation by using a suitable variable separation ansatz. For the BKK system, the related trilinear equation (9) can be solved by the following variable separation ansatz:

$$f=a_0+a_1p(x,t)+a_2q(y,t)+a_3p(x,t)q(y,t) \qquad (12)$$

where a_0, a_1, a_2 and a_3 are arbitrary constants and $p\equiv p(x,t)$ and $q\equiv q(y,t)$ are functions of $\{x,t\}$ and $\{y,t\}$, respectively. It is clear that the variables x and y now have been separated totally.

Substituting (12) into (9) with (11) yields

$$\left[-2(a_1+a_3q)+fp_x^{-1}\partial_x\right](p_t+2h_0p_x+p_{xx})+\left[-2(a_2+a_3p)+fq_y^{-1}\partial_y\right]q_t=0. \qquad (13)$$

Because p is y independent and q is x independent, equation (13) can be separated into two equations,

$$p_t=-p_{xx}-2h_0p_x+(a_0a_3-a_1a_2)(c_1p^2-c_3p+c_2) \qquad (14)$$

$$q_t=c_1(a_0+a_2q)^2+c_2(a_1+a_3q)^2+c_3(a_0+a_2q)(a_1+a_3q) \qquad (15)$$

where the arbitrary functions $c_1\equiv c_1(t)$, $c_2\equiv c_2(t)$, $c_3\equiv c_3(t)$ are introduced by the variable separation procedure.

Finally, because the function h_0 is an arbitrary function of $\{x,t\}$, solving the variable separation equation (14) becomes a trivial trick. One can treat the function p as an arbitrary function while the function $h_0(x,t)$ can be fixed by (14), i.e.

$$h_0=\frac{1}{2p_x}[(a_0a_3-a_1a_2)(c_1p^2-c_3p+c_2)-p_t-p_{xx}]. \qquad (16)$$

In the same way, because the functions c_1, c_2 and c_3 are arbitrary functions of t, one can easily find some quite general solutions of the Riccati equation (15). Here are two special examples.

(1) If we write c_1, c_2 and c_3 as

$$c_1=\frac{a_3^2A_{2t}}{(a_1a_2-a_0a_3)^2}-\frac{a_3(a_1+a_3A_2)A_{1t}}{(a_1a_2-a_0a_3)^2A_1}-\frac{(a_1+a_3A_2)^2A_{3t}}{(a_1a_2-a_0a_3)^2A_1} \qquad (17)$$

$$c_2=\frac{a_2^2A_{2t}}{(a_1a_2-a_0a_3)^2}-\frac{a_2(a_0+a_2A_2)A_{1t}}{(a_1a_2-a_0a_3)^2A_1}-\frac{(a_0+a_2A_2)^2A_{3t}}{(a_1a_2-a_0a_3)^2A_1} \qquad (18)$$

$$c_3 = \frac{(a_0a_3 + a_1a_2 + 2a_2a_3A_2)A_{1t}}{(a_1a_2 - a_0a_3)^2A_1} - \frac{2a_2a_3A_{2t}}{(a_1a_2 - a_0a_3)^2} + 2\frac{(a_0 + a_2A_2)(a_1 + a_3A_2)A_{3t}}{(a_1a_2 - a_0a_3)^2A_1} \tag{19}$$

with $A_1 \equiv A_1(t)$, $A_2 \equiv A_2(t)$ and $A_3 \equiv A_3(t)$ being arbitrary functions of t, then the general solution of (15) with (17)–(19) reads

$$q = \frac{A_1}{A_3 + F_1(y)} + A_2 \tag{20}$$

where $F_1 \equiv F_1(y)$ is an arbitrary function of y.

(2) If we select c_1, c_2 and c_3 as

$$c_1 = \frac{a_3^2b_{0t}}{(a_1a_2 - a_0a_3)^2} - \frac{a_3(a_1 + a_3b_0)b_{1t}}{(a_1a_2 - a_0a_3)^2b_1} - \frac{\left[(a_1 + a_3b_0)^2 - b_1^2a_3^2\right]b_{2t}}{(a_1a_2 - a_0a_3)^2b_1} \tag{21}$$

$$c_2 = \frac{a_2^2b_{0t}}{(a_1a_2 - a_0a_3)^2} - \frac{a_2(a_0 + a_2b_0)b_{1t}}{(a_1a_2 - a_0a_3)^2b_1} - \frac{\left[(a_0 + a_2b_0)^2 - a_2^2b_1^2\right]b_{2t}}{(a_1a_2 - a_0a_3)^2b_1} \tag{22}$$

$$c_3 = \frac{(a_0a_3 + a_1a_2 + 2a_2a_3b_0)b_{1t}}{(a_1a_2 - a_0a_3)^2b_1} - \frac{2a_2a_3b_{0t}}{(a_1a_2 - a_0a_3)^2} + 2\frac{\left[(a_0 + a_2b_0)(a_1 + a_3b_0) - a_2a_3b_1^2\right]b_{2t}}{(a_1a_2 - a_0a_3)^2b_1} \tag{23}$$

with $b_0 \equiv b_0(t)$, $b_1 \equiv b_1(t)$ and $b_2 \equiv b_2(t)$ being arbitrary functions of t, then the general solution of (15) with (21)–(23) reads

$$q = b_1 \tanh(b_2 + F_2(y)) + b_0 \tag{24}$$

with $F_2 \equiv F_2(y)$ being an arbitrary function of y.

Now, substituting (12) into (8) with (11), we find that the BKK equation possesses an exact solution

$$H = \frac{(a_1 + a_3q)p_x}{a_0 + a_1p + a_2q + a_3pq} + h_0 \tag{25}$$

$$G = \frac{p_xq_y(a_0a_3 - a_1a_2)}{(a_0 + a_1p + a_2q + a_3pq)^2} \tag{26}$$

where p is an arbitrary function, q is given by (20) or (24) or any other solutions of (15) while h_0 is determined by (16).

Because of the arbitrariness of the functions p and q, one can find abundant localized excitations for the quantity G of the BKK system. Actually, expression (26) is valid for various (2 + 1)-dimensional models such as the NNV (Nizhnik–Novikov–Veselov) equation, ANNV (asymmetric NNV) equation, DS (Davey–Stewartson) equation, ADS (asymmetric DS) equation [15], dispersive long wave equation (DLWE) and a general $(N + M)$-component AKNS system [16–20]. Many kinds of interesting localized excitations have been discussed in our previous papers [16–20]. In the next section, we focus our attention on the possible compacton structures and especially on the interaction behaviour of compactons.

3. Compacton solutions and their interaction behaviour

It is known that in the (1 + 1)-dimensional case, different from other types of solitary wave solutions, the compacton solutions are completely supported in a small finite region. Outside this region, the compacton possesses the zero value identically. To my knowledge, there are many papers studying the (1 + 1)-dimensional compactons, however, there is no paper discussing whether any type of compacton solution that is localized in all directions exists in higher dimensions. Actually, similar to the continuous soliton-like solutions, there may be quite rich compacton solutions in high dimensions because some arbitrary functions can be included in the solutions of the high-dimensional models.

For the BKK system, the compacton solutions for the field G can be found simply by selecting the arbitrary functions in expression (26) as piecewise smooth functions, say, if we take

$$p = \sum_{i=1}^{N} \begin{cases} 0 & x + v_i t \leqslant x_{1i} \\ p_i(x + v_i t) - p_i(x_{1i}) & x_{1i} < x + v_i t \leqslant x_{2i} \\ p_i(x_{2i}) - p_i(x_{1i}) & x + v_i t > x_{2i} \end{cases} \tag{27}$$

and

$$q = \sum_{j=1}^{M} \begin{cases} 0 & y \leqslant y_{1j} \\ q_j(y) - q_j(y_{1j}) & y_{1j} < y \leqslant y_{2j} \\ q_j(y_{2j}) - q_j(y_{1j}) & y > y_{2j} \end{cases} \tag{28}$$

where the functions $p_i, q_j, i = 1, 2, \ldots, N, j = 1, 2, \ldots, M$ are all arbitrary differentiable functions with the conditions

$$p_{ix}|_{x=x_{1i}} = p_{ix}|_{x=x_{2i}} = 0, \qquad q_{jy}|_{y=y_{1j}} = q_{jy}|_{y=y_{2j}} = 0 \tag{29}$$

then expression (26) becomes some type of (2 + 1)-dimensional multiple compacton solution of the BKK system. In the selection (28), the functions A_i and c_i of (17)–(20) are fixed as

$$A_1 = 1 \qquad A_2 = A_3 = c_1 = c_2 = c_3 = 0 \qquad q = \frac{1}{F_1(y)}. \tag{30}$$

For a (1 + 1)-dimensional nonlinear equation

$$F(u, u_x, u_t, u_{xx}, \ldots) = 0 \tag{31}$$

the compacton solutions are weak ones. That means that though the (1 + 1)-dimensional compacton solutions are non-differentiable, substituting them into (31) really yields a zero distribution. In (2 + 1) dimensions, the situation is quite different. The satisfaction of the nonlinear equation is guaranteed by the separation (14) and (15). So in principle, to find some types of exact solutions, it is not necessary to put the once differentiable conditions (introduced by (29)) on the functions p and q. However, similar to the (1 + 1)-dimensional case, we hope some of the physical quantities, say, G for the BKK system, will still be continuous anywhere and anytime. Because of the entrance of the first-order derivatives of the functions p and q in the expression for G (26), we require the functions p and q to be once differentiable.

To study the interaction properties of the compactons, we should study the limiting procedures, $t \to \mp\infty$, for (26) with (27) and (28). Without loss of generality, we assume that

$$v_1 < v_2 < \cdots < v_N \tag{32}$$

for convenience of discussion. Because of the zero value property of the ith compacton outside its supported region, we can write down its *exact* expressions before and after interaction.

Before interaction ($t \to -\infty$), the ith compacton can be expressed by

$$G_i^- = \frac{(a_0 a_3 - a_1 a_2) P_{ix} q_y}{(a_0 + a_1 P_i + a_2 q + a_3 P_i q)^2} \tag{33}$$

where q is still given by (28) and P_i is related to the original p_i of (27) by

$$P_i^- = \sum_{j<i} [p_j(x_{2j}) - p_j(x_{1j})] + \begin{cases} 0 & x + v_i t \leqslant x_{1i} \\ p_i(x + v_i t) - p_i(x_{1i}) & x_{1i} < x + v_i t \leqslant x_{2i} \\ p_i(x_{2i}) - p_i(x_{1i}) & x + v_i t > x_{2i} \end{cases} \tag{34}$$

while the general multiple compacton solution (26) can be written as

$$G_{t\to-\infty} = \sum_{i=1}^{N} G_i^- . \tag{35}$$

In the same way, after the interaction ($t \to +\infty$), we have

$$G_{t\to+\infty} = \sum_{i=1}^{N} G_i^+ \tag{36}$$

$$G_i^+ = \frac{(a_0a_3 - a_1a_2)P_{ix}^+ q_y}{\left(a_0 + a_1P_i^+ + a_2q + a_3P_i^+ q\right)^2} \tag{37}$$

$$P_i^+ = \sum_{j>i}[p_j(x_{2j}) - p_j(x_{1j})] + \begin{cases} 0 & x + v_i t \leqslant x_{1i} \\ p_i(x + v_i t) - p_i(x_{1i}) & x_{1i} < x + v_i t \leqslant x_{2i} \\ p_i(x_{2i}) - p_i(x_{1i}) & x + v_i t > x_{2i}. \end{cases} \tag{38}$$

Remark 1. Expressions (33)–(38) are exactly correct and not approximate.

Remark 2. For the concrete examples, it is not necessary to take $t \to \infty$ in the limiting procedures. Expression (35) is always correct before interactions (all the compactons have not yet met) and (36) is always valid after interactions (all the compactons have passed through each other).

Remark 3. There is no phase shift for the interactions of the compactons expressed by (26) with (27) and (28).

From the limiting results (33) with (34) and (37) with (38), we can see that the interactions among compactons expressed by (26) with (27) and (28) are nonelastic for the general selections of p_i and q_j with

$$p_i(x_{2i}) - p_i(x_{1i}) \neq 0 \quad \text{at least for one of } i. \tag{39}$$

However, if we appropriately select p_i such that

$$p_i(x_{2i}) - p_i(x_{1i}) = 0 \quad \forall i \tag{40}$$

then the interactions among these types of compactons become completely elastic!

To see the interaction behaviour of the compactons more concretely and visually, we discuss two special examples by fixing the arbitrary functions p_i and q_j further.

Example 1 (Compactons without elastic interaction behaviour)**.** One of the simplest selections of the compactons without elastic interaction behaviour may be

$$p = \sum_{i=1}^{N} \begin{cases} 0 & x + v_i t \leqslant x_{0i} - \frac{\pi}{2k_i} \\ b_i \sin(k_i(x + v_i t - x_{0i})) + b_i & x_{0i} - \frac{\pi}{2k_i} < x + v_i t \leqslant x_{0i} + \frac{\pi}{2k_i} \\ 2b_i & x + v_i t > x_{0i} + \frac{\pi}{2k_i} \end{cases} \tag{41}$$

and

$$q = \sum_{j=1}^{M} \begin{cases} 0 & y \leqslant y_{0j} - \frac{\pi}{2l_j} \\ d_j \sin(l_j(y - y_{0j})) + d_j & y_{0j} - \frac{\pi}{2l_j} < y \leqslant y_{0j} + \frac{\pi}{2l_j} \\ 2d_j & y > y_{0j} + \frac{\pi}{2l_j} \end{cases} \tag{42}$$

where $b_i, k_i, v_i, x_{0i}, d_j, l_j$ and y_{0j} are all arbitrary constants.

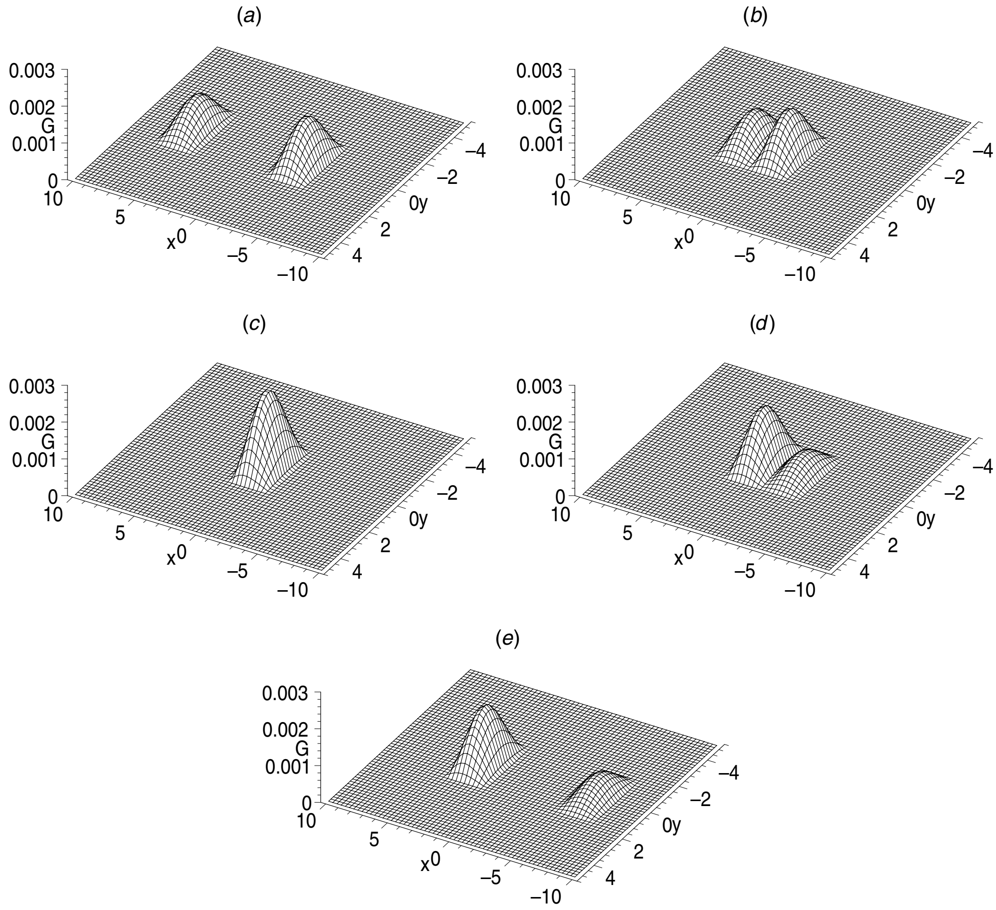

Figure 1. Evolution plot of a two-compacton solution (26) with (41), (42) and (43) at times (*a*) $t = -3$, (*b*) $t = -1$, (*c*) $t = 0$, (*d*) $t = 1$, (*e*) $t = 3$.

Figure 1 is the evolution plot of a two-compacton solution (26) with (41), (42) and

$$N = 2 \qquad M = 1 \qquad a_0 = 20 \qquad a_1 = a_2 = 25a_3 = 1 \qquad b_1 = -2 \qquad v_1 = -1$$
$$-b_2 = d_1 = k_1 = k_2 = l_1 = 1 \qquad x_{01} = x_{02} = y_{01} = 0 \qquad v_2 = 2. \tag{43}$$

In [21] and [20], it has been pointed out that the interaction between two travelling ring shape soliton solutions is completely elastic and the interaction between two travelling peakons is not completely elastic, two peakons may completely exchange their shapes. From figure 1, we see that the interaction between two compactons (which possess the property (39)) exhibits a new phenomenon. The interaction is non-elastic but two compactons do not completely exchange their shapes. We have also graphically checked the correctness of expressions (33)–(34) for the selections (41), (42) and (43) at $t = -3$ (before interaction) and $t = 3$ (after interaction).

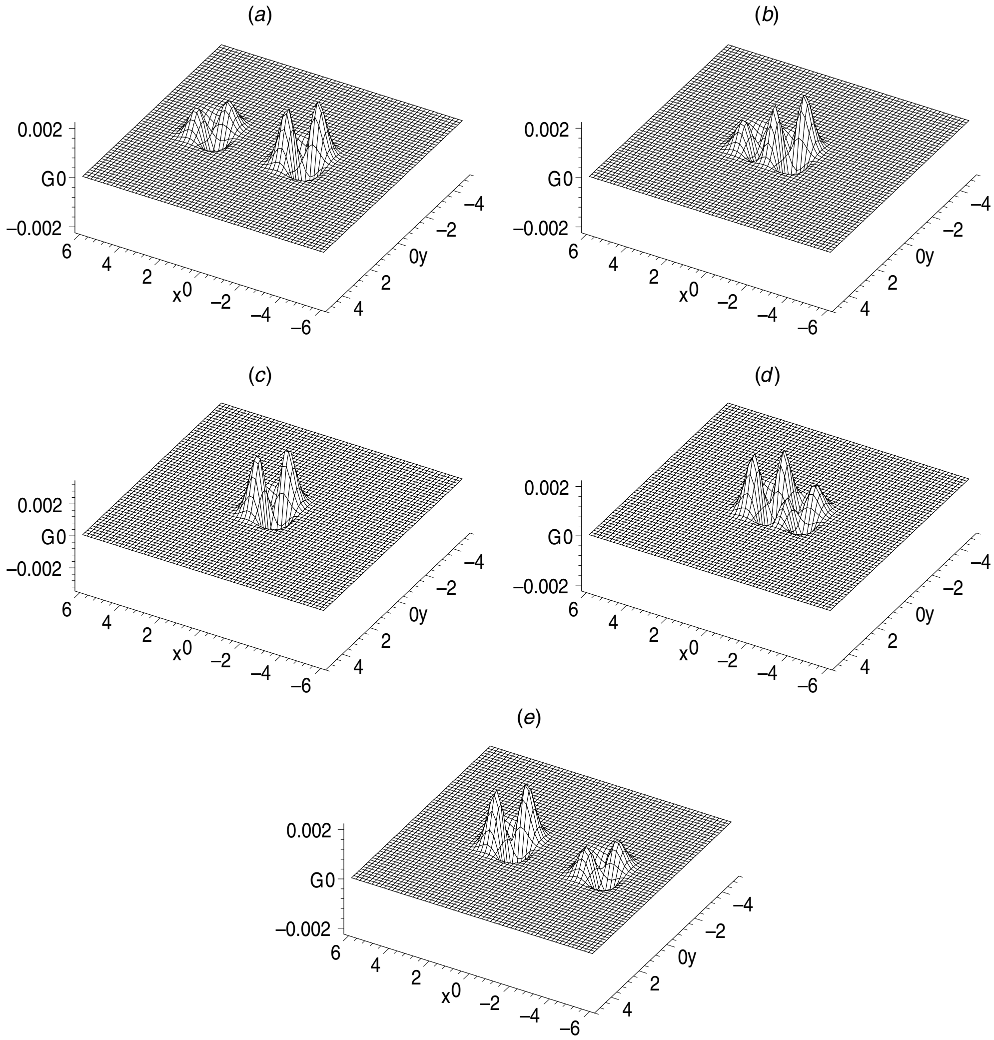

Figure 2. Evolution plot of a two-compacton solution (26) with (44), (45) and (46) at times (*a*) $t = -1.5$, (*b*) $t = -0.5$, (*c*) $t = 0$, (*d*) $t = 0.5$, (*e*) $t = 1.5$.

Example 2 (Compactons with completely elastic interaction behaviour)**.** One of the simplest selections of the compactons with elastic interaction behaviour may be

$$p = \sum_{i=1}^{N} \begin{cases} 0 & x + v_i t \leqslant x_{0i} - \frac{\pi}{2k_i} \\ b_i \cos^{\alpha_i+1}(k_i(x + v_i t - x_{0i})) & x_{0i} - \frac{\pi}{2k_i} < x + v_i t \leqslant x_{0i} + \frac{\pi}{2k_i} \\ 0 & x + v_i t > x_{0i} + \frac{\pi}{2k_i} \end{cases} \tag{44}$$

and

$$q = \sum_{j=1}^{M} \begin{cases} 0 & y \leqslant y_{0j} - \frac{\pi}{2l_j} \\ d_j \cos^{\beta_j+1}(l_j(y - y_{0j})) & y_{0j} - \frac{\pi}{2l_j} < y \leqslant y_{0j} + \frac{\pi}{2l_j} \\ 0 & y > y_{0j} + \frac{\pi}{2l_j} \end{cases} \tag{45}$$

where $b_i, k_i, v_i, x_{0i}, d_j, l_j$ and y_{0j} are all arbitrary constants and $\{\alpha_i, \beta_j\}$ for all $\{i, j\}$ are positive integers.

Figure 2 shows the completely elastic interaction property of a two-compacton solution (26) with (44), (45) and

$$N = 2 \qquad M = 1 \qquad a_0 = 20 \qquad a_1 = a_2 = 25a_3 = 1 \qquad b_1 = -2 \qquad v_1 = -1$$
$$v_2 = 2 \qquad -b_2 = d_1 = k_1 = k_2 = l_1 = 1 \qquad x_{01} = x_{02} = y_{01} = 0$$
$$\alpha_1 = \alpha_2 = \beta_1 = 4. \tag{46}$$

The correctness of expressions (37)–(38) for the selections (44), (45) and (46) at $t = -1.5$ (before interaction) and $t = 1.5$ (after interaction) have also been graphically checked.

4. Summary and discussion

In summary, in (2 + 1) dimensions, there are quite rich localized excitations. In a series of our previous papers [16–23], many types of continuous localized solutions such as dromions, lumps, breathers, ring solitons and some types of piecewise peakon solutions have been obtained. Recently, many scientists have studied another type of (1 + 1)-dimensional piecewise excitation, compactons. In this short paper, some types of (2 + 1)-dimensional compacton solutions that are localized in all directions have been given.

Though the travelling dromions and the travelling saddle-type ring soliton solutions possess completely elastic interaction properties [20, 21, 23], the interactions between two travelling compacton solutions may have quite rich behaviour. For the general selections (27) and (28) with (39), the interactions are non-elastic. The shapes of these types of compactons will be changed after their interactions. For the special selections (27) and (28) with (40), the interactions among compactons become completely elastic.

Because expression (26) is valid for many (2 + 1)-dimensional integrable models [20], the compacton excitations and their interaction behaviour discussed here may be quite universal in higher dimensions. Though the (1 + 1)-dimensional compacton solutions have been studied widely in the literature [2–5], to our knowledge, the higher dimensional compacton solutions localized in all directions are first found in this paper. So the more is said about the higher dimensional compacton solutions the more they are worthy of further study.

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